Reflection vs Persuasion. Modeling Opinion Formation in a Society

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Opinion Dynamic

General Idea

$$\text{Person} \Rightarrow \text{Opinion}$$

Opinion Dynamic

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Person $\Rightarrow$ Opinion

Influence

Opinion Dynamic

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Reflection

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Opinion Dynamic

Goal:

Describe the agreement-disagreement dynamic between the individuals of a group with their **complexities**.

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¿How?

- Opinion states.
- Transition processes.

Preliminaries

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- Social analysis $\Rightarrow$ groups.
- People interact.
- Various views with varying degrees of conviction.
- Gradual changes in opinion.
- Simplicity of the model vs Real system.

Analytical Model

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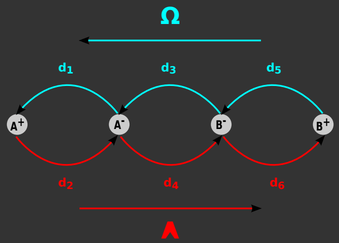
Interaction



Cuadratic order

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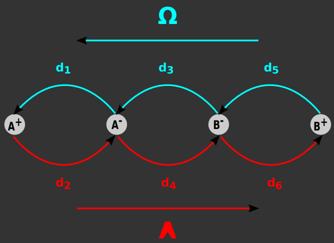
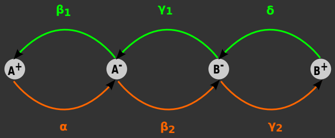
Interaction



Cuadratic order

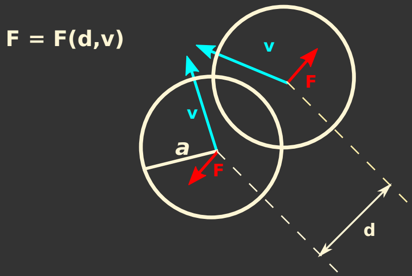
Reflection/Mass media



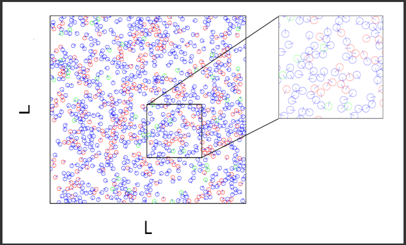
Linear order

Computational Model

- Self-propelled agent-model (constant velocity).
- Low densities ⇒ Binary collisions.



Interaction: Soft core potential.



(Peruani and Sibona 2008)

Correction of probabilities.

Prob. of change by Interaction

$$f'_{ij} = \frac{1}{\mu + \tau}\left[\frac{f_{ij}}{\Sigma_{ij}}\left(1 - e^{-\Sigma_{ij}\mu(v)}\right)\right]$$

Prob. of change by Reflection

$$r'_{ij} = \frac{\tau}{\mu + \tau}(r_{ij})$$

Donde:

- f_{ij}: change frequency by interaction to right (λ_{ij}) or to left (ω_{ij})
- $\Sigma_{ij} = \omega_{ij} + \lambda_{ij} + \omega_{ji} + \lambda_{ji}$ Transition flow.
- r_{ij}: change frequency by reflection to right $(\alpha,\ \beta_2,\ \gamma_2)$ or to left $(\delta,\ \beta_1,\ \gamma_1)$
- $\mu(v) = 1{,}18(1/v)^{0{,}967}$ $(a = 1)$ Mean duration time of a collision. (Terranova et al.)
- $\tau = 1/\sigma_0 \rho v$ $(\sigma_0 = 4a)$ Mean free time

Correction of probabilities

Choosing $\lambda_{ij} = \lambda, \quad \omega_{ij} = \omega \Rightarrow$ Exact solution.

We can write:

$$f'_{ij} + r'_{ij} = \frac{1}{\sigma_0 \rho v \mu + 1} \left\{ \sigma_0 \rho v \left[\frac{f_{ij}}{\Sigma_{ij}} \left(1 - e^{-\Sigma_{ij} \mu(v)} \right) \right] + r_{ij} \right\}$$

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Limit situations:

- $v >> \Sigma_{ij}$ $(v \to \infty)$ $\quad \Rightarrow \quad f'_{ij} + r'_{ij} = \sigma_0 \rho f_{ij} + r_{ij}$
- $v << \Sigma_{ij}$ $(v \to 0)$ $\quad \Rightarrow \quad f'_{ij} + r'_{ij} = \frac{\sigma_0 \rho v}{\Sigma_{ij}} f_{ij} + r_{ij}$

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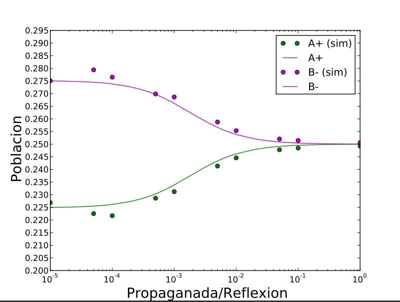
At equilibrium:

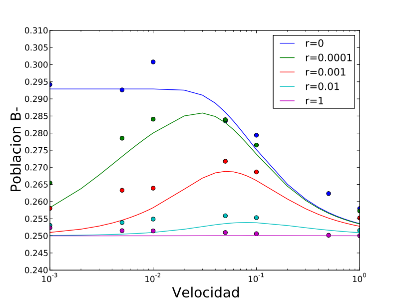
- $\frac{f'_{ij}}{r'_{ij}} >> 1 \quad \Rightarrow$ Interaction dominates
- $\frac{f'_{ij}}{r'_{ij}} << 1 \quad \Rightarrow$ Reflection/mass media dominates

Model Validation

Population vs Reflection

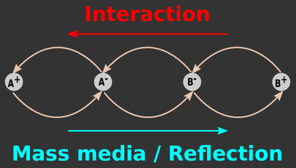


Population vs Velocity



$\lambda_{ij} = \omega_{ij} = 0,05$ (interaction), $r_{ij} = r$ (ref./mass media = cte)

Interaction vs Mass Media/Reflection



$$\lambda_{ij} = 0,05\ ,\ \ \omega_{ij} = 0,1$$

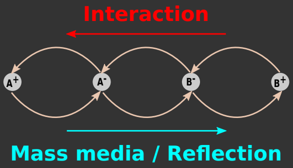
$$\Sigma_{ij} = 0,3$$

$$r_2 = \alpha,\ \beta_2,\ \gamma_2$$

$$r_1 = \delta,\ \beta_1,\ \gamma_1$$

$$r_2 > r_1$$

Interaction vs Mass Media/Reflection



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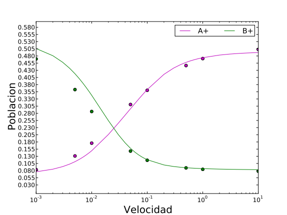
$$r_2 > r_1$$

$$r_1 = 0,0003 \ r_2 = 0,0006$$

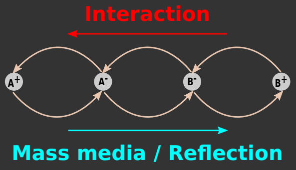


- $v >> \Sigma_{ij}$ we have $\frac{f'_{ij}}{r'_{ij}} >> 1$

$$\therefore P_{B^+} < P_{B^-} < P_{A^-} < P_{A^+}$$

Interaction dominates.

Interaction vs Mass Media/Reflection



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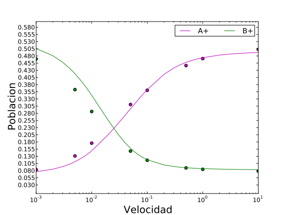

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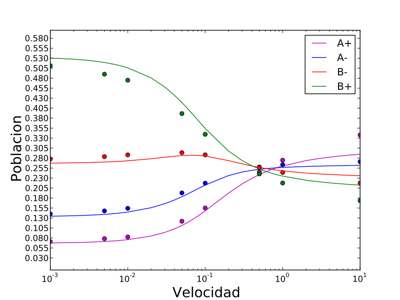
  $\therefore P_{B^+} < P_{B^-} < P_{A^-} < P_{A^+}$
  Interaction dominates.

- $v << \Sigma_{ij}$ we have $\frac{f'_{ij}}{r'_{ij}} << 1$

  $\therefore P_{B^+} > P_{B^-} > P_{A^-} > P_{A^+}$
  Reflection/mass media dominates.

Interaction vs Mass Media/Reflection

$r_1 = 0,003 \; r_2 = 0,006$



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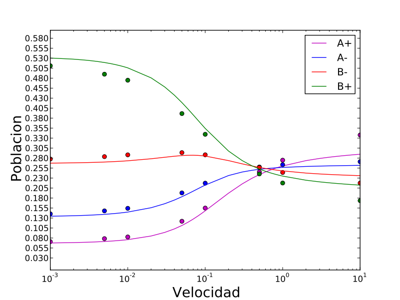

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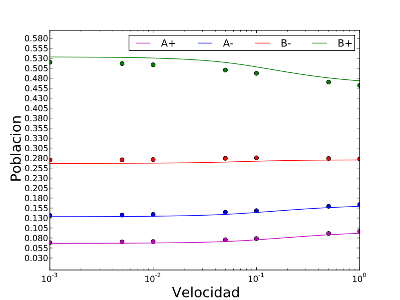

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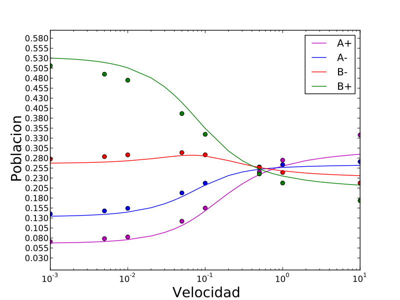

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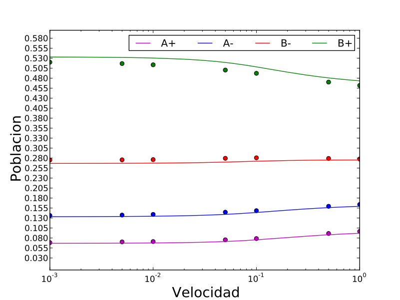

Not cross in $r_1 = 0,03 \; r_2 = 0,06$:

$$v >> \Sigma_{ij} \text{ now } \frac{f'_{ij}}{r'_{ij}} << 1$$

$$\therefore P_{B^+} > P_{B^-} > P_{A^-} > P_{A^+}$$

Reflection/mass media **ALWAYS** dominates.

Conclusions

- Novel model with an important analytical development.

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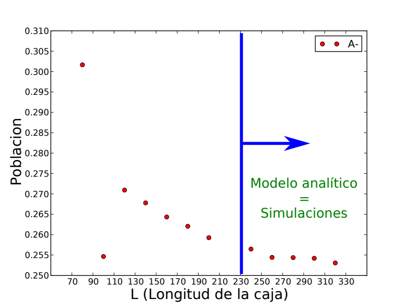

$$v = 0,1, \lambda = \omega = 0,05, \; r = 0,01$$

Where and why parallelize?

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- Interaction force calculation.
- Exchange of opinion.

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Why?

Reduce simulation times

Peruani, F. and G. Sibona (2008).
Dynamics and Steady States in Excitable Mobile Agent Systems.

Physical Review Letters 100.

Terranova, G., J. Revelli, and G. Sibona (2012).
Opinion Formation model for Interacting Self-propelled Agents.
En preparación.